

Surname and name \_\_\_\_\_

Lesson 11, 12 and 13. Similarity. Thales. Pythagoras. Solids. Areas and volumes A

1. A cuboid shaped package is 8 cm long, 25 cm high and 5 cm wide.

a) Calculate the total surface area.

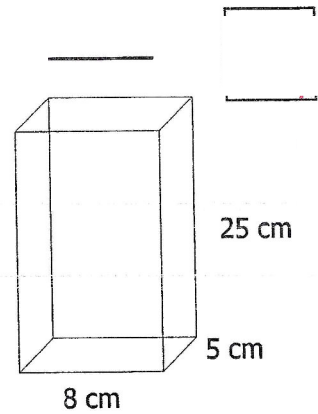
b) Calculate the volume.

We make a similar package 10 cm long.

c) What is the similarity ratio between the first package and the new one?

d) Calculate the dimensions for this new package, that is, the height and the width.

e) Calculate the surface ratio and the volume ratio.



$$a) A_T = 2(a \cdot b) + 2(b \cdot c) + 2(a \cdot c) = 2 \cdot 8 \cdot 5 + 2 \cdot 5 \cdot 25 + 2 \cdot 25 \cdot 8 = 80 + 250 + 400 = 730 \text{ cm}^2$$

$$b) V = a \cdot b \cdot c = 8 \cdot 5 \cdot 25 = 1000 \text{ cm}^3$$

$$c) r = \frac{a'}{a} = \frac{10}{8} = 1.25$$

$$d) 1.25 = \frac{b'}{5} = \frac{c'}{25}; \quad b' = 1.25 \cdot 5 = 6.25 \text{ cm is the new wide}$$

$$c' = 1.25 \cdot 25 = 31.25 \text{ cm is the new high.}$$

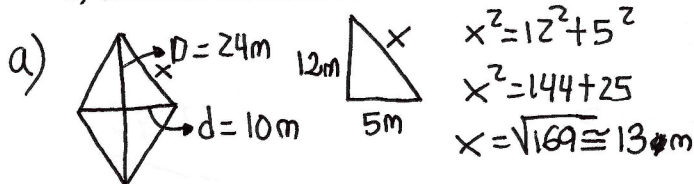
$$e) r^2 = 1.25^2 = 1.56$$

$$r^3 = 1.25^3 = 1.95$$

2. The diagonals of a rhombus are  $d = 10 \text{ m}$  and  $D = 24 \text{ m}$ .

a) Calculate the perimeter.

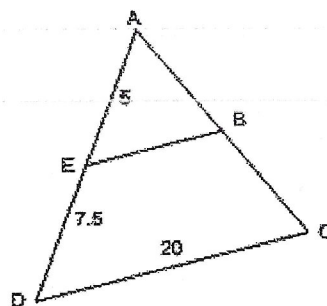
b) Calculate the surface area.



$$P = 4 \cdot \ell = 4 \cdot 13 = 52 \text{ m.}$$

$$b) A = \frac{D \cdot d}{2} = \frac{24 \cdot 10}{2} = \frac{240}{2} = 120 \text{ m}^2$$

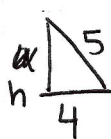
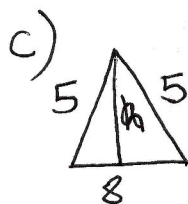
3. The drawing shows two similar triangles with a common vertex A.
- Determine the length of EB.
  - What is the similarity ratio between two triangles?
  - Knowing that  $AB=AE$ , that is, the triangle AEB is isosceles, what is the area of this triangle?
  - And finally, what is the area for the largest triangle. That is ADC triangle.



a)  $\frac{20}{12.5} = \frac{EB}{5}$ ;  $EB = \frac{20 \cdot 5}{12.5} = \frac{100}{12.5} = 8$  is the length

$AD = 5 + 7.5 = 12.5$

b)  $r = \frac{20}{12.5} = \frac{8}{5}$  is the similarity ratio  $\frac{20}{8} = 2.5$

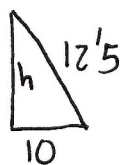
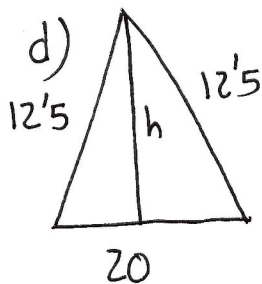


$$5^2 = 4^2 + h^2$$

$$25 = 16 + h^2$$

$$h = \sqrt{25 - 16} \cong 3$$

$$A = \frac{b \cdot h}{2} = \frac{8 \cdot 3}{2} = \frac{24}{2} = 12 u^2$$



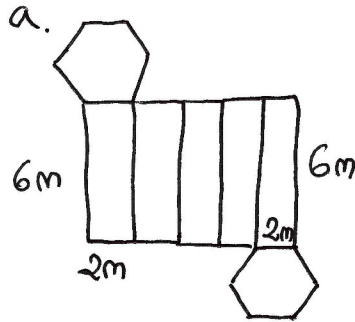
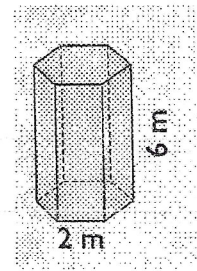
$$12.5^2 = h^2 + 10^2$$

$$156.25 = h^2 + 100$$

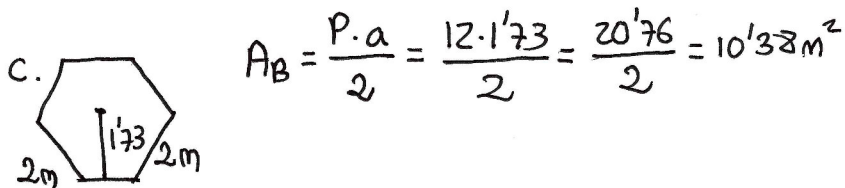
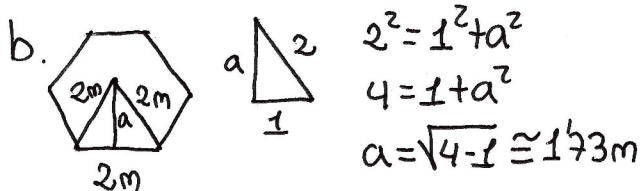
$$h = \sqrt{156.25 - 100} \cong 7.5$$

$$A = \frac{b \cdot h}{2} = \frac{20 \cdot 7.5}{2} = \frac{150}{2} = 75 u^2$$

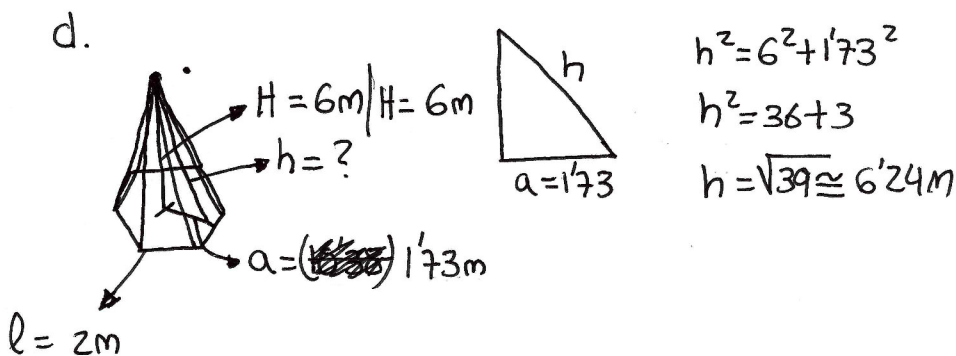
4. This is a hexagonal prism whose height is 6 m and the base edge is 2 m.
- Draw the unfolding and calculate the lateral area.
  - Calculate the base apothem.
  - Calculate the base area.
  - Calculate the apothem for the hexagonal pyramid that has the same base and the same height as this prism.



$$A_L = 6 \cdot b \cdot a; A_L = 6 \cdot 2 \cdot 6 = 72 \text{ m}^2$$



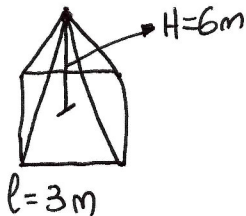
$$P = 6 \cdot l = 6 \cdot 2 = 12 \text{ m}$$



5. A quadrangular pyramid has 3 m base edge and 6 m height.

- Draw the pyramid and write down the measures on it.
- Calculate the base apothem.
- Calculate the pyramid apothem.
- Calculate the total surface area.

a)

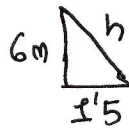
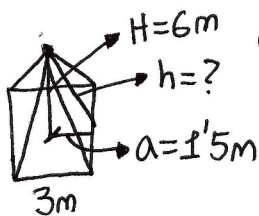


b)

$$a = \frac{l}{2} = \frac{3}{2} = 1.5m$$

A hand-drawn diagram of a square base with side length 3m. A line from the center to the midpoint of one side is labeled  $a$ .

c)



$$h^2 = c^2 + c^2$$

$$h^2 = 6^2 + 1.5^2$$

$$h^2 = 36 + 2.25$$

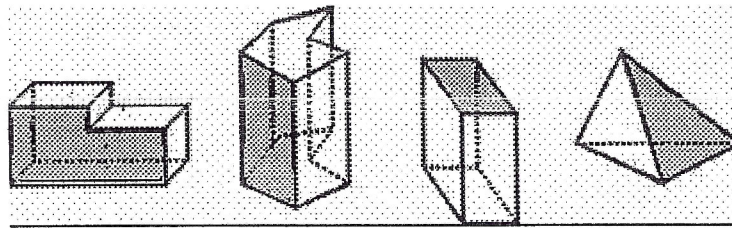
$$h = \sqrt{38.25} \approx 6.18m$$

d)  $A_T = A_B + A_L$   $A_T = 9 + 37.08 = 46.08m^2$

$$A_B = l^2 = 3^2 = 9m^2$$

$$A_L = 4 \cdot \frac{b \cdot h}{2} = 4 \cdot \frac{3 \cdot 6.18}{2} = 4 \cdot 9.27 = 37.08m^2$$

6. Classify the following polyhedrons:



A

B

C

D

a) You have to write: concave or convex; regular or irregular.

A: concave/irregular

B: concave/irregular

C: convex/regular

D: convex/irregular

b) What kind of polyhedron are B, C and D respectively?

B: prisma (~~recta~~) hexagonal irregular, recto, octaedro

C: ortoedro rectangular, recto, regular, hexaedro.

D: tetraedro triangular, irregular.

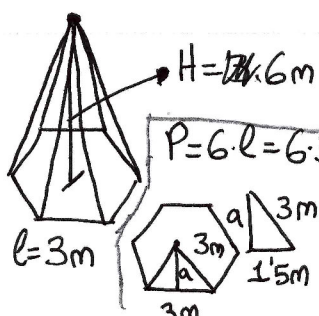
c) Complete the table and verify Euler's formula?

	Nº of edges	Nº of faces	Nº of vertices	Euler's formula
A	18	8	12	$C+V=A+2; 8+12=18+2;$ $20=20 \checkmark$
B	18	8	12	$C+V=A+2; 8+12=18+2;$ $20=20 \checkmark$
C	12	6	8	$C+V=A+2; 8+8=12+2;$ $14=14 \checkmark$
D	6	4	4	$C+V=A+2; 4+4=6+2;$ $8=8 \checkmark$



7. Work out the volume for a hexagonal pyramid whose base edge is 3m and its height is 6m. Give your result to two decimal places.

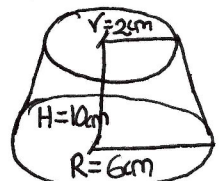
$$V = \frac{1}{3} A_B \cdot H; V = \frac{1}{3} 23'4 \cdot 6 = 7'8 \cdot 6 = \underline{\underline{46'8 \text{ m}^3}}$$



$H = 6 \text{ m}$   
 $l = 3 \text{ m}$   
 $P = 6 \cdot l = 6 \cdot 3 = 18 \text{ m}$   
 $A_B = \frac{P \cdot a}{2} = \frac{18 \cdot 2'6}{2} = 23'4 \text{ m}^2$   
 $3^2 = 1'5^2 + a^2$   
 $9 = 2'25 + a^2$   
 $a^2 = 9 - 2'25$   
 $a = \sqrt{6'75} \approx 2'6 \text{ m}$

8. Calculate the lateral area, surface area and volume of a **truncated cone** with radii of 2 and 6 cm and a height of 10 cm.

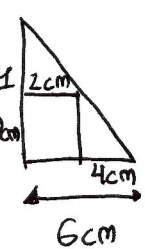
**Area lateral:**  
 $A_L = \pi(R+r)G = \pi(6+2)10'77 = \pi 8 \cdot 10'77 = 86'16\pi = \underline{\underline{270'68 \text{ cm}^2}}$



$G^2 = 10^2 + 4^2$   
 $G^2 = 100 + 16$   
 $G = \sqrt{116} \approx 10'77 \text{ cm}$

**Area total:**  $A_T = A_B + A_b + A_L$   $A_T = 113'1 + 12'57 + 270'68 = \underline{\underline{396'35 \text{ cm}^2}}$

$A_B = \pi \cdot R^2 = \pi \cdot 6^2 \approx 113'1 \text{ cm}^2$   
 $A_b = \pi \cdot r^2 = \pi \cdot 2^2 \approx 12'57 \text{ cm}^2$   
 $A_L = 270'68 \text{ cm}^2$



$\frac{h_1}{H} = \frac{r}{R-r}; \frac{h_1}{10} = \frac{2}{6-2};$   
 $h_1 = \frac{10 \cdot 2}{4} = \frac{20}{4} = 5 \text{ cm}$

**Volumen:**  $V_T = V_{cm} - V_{cm}$   $V_T = 565'5 - 20'95 = \underline{\underline{544'55 \text{ cm}^3}}$

$V_{cm}: \text{altura} = 10 + 5 = 15 \text{ cm}$   
 $A_B = \pi \cdot R^2 = \pi \cdot 6^2 = 113'1 \text{ cm}^2$

$V_{cm} = \frac{1}{3} A_B \cdot H = \frac{1}{3} 113'1 \cdot 15 = 37'7 \cdot 15 = 565'5 \text{ cm}^3$

$V_{cm}: \text{altura} = 5 \text{ cm}$

$V_{cm} = \frac{1}{3} A_b \cdot h_1 = \frac{1}{3} 12'57 \cdot 5 = 4'19 \cdot 5 = 20'95 \text{ cm}^3$

$A_b = \pi \cdot r^2 = \pi \cdot 2^2 = 12'57 \text{ cm}^2$

9. a) What does Thales theorem state?

Parallels through several straight lines form proportional segments.

b) Write the definition for each word which is referred to a pyramid

1. Apex.

The point where the lateral faces join in a pyramid.

2. Edge.

The segment that is between two consecutive faces.

3. Dihedron angle.

The separation between two planes with a common line.

4. Apothem.

The height of its lateral faces.

c) Could you write the surfaces you obtain by unfolding a frustum of a quadrilateral pyramid?

Two squares and four isosceles trapeziums.

d) What is a cuboid?

It is a polyhedron whose faces are perpendicular in every edge.